APPENDIX H

FILTER DESIGN MATERIAL

This appendix provides filter design tables and circuits that supplement the material presented in Chapter 14.
Filter Type and $T(s)$

(a) Low pass (LP)

\[ T(s) = \frac{a_0}{s + \omega_0} \]

(b) High pass (HP)

\[ T(s) = \frac{a_1 s}{s + \omega_0} \]

(c) General

\[ T(s) = \frac{a_0 s + a_1}{s + \omega_0} \]

$s$-Plane Singularities

- Pole at $\omega_0$
- Zero at $\omega_0$
| $T(s)$ | Singularities | $|T|$ and $\phi$ | Passive Realization | Op Amp–RC Realization |
|-------|--------------|----------------|-------------------|---------------------|
| All pass (AP) |

$T(s) = -a_1 \frac{s - \omega_0}{s + \omega_0}$

$a_1 > 0$

$20 \log |a_1|$

$CR = 1/\omega_0$

Flat gain ($a_1$) = 0.5

\[ \frac{|V_o|}{|V_i|} = 1 \]

$\phi(\omega) = -2 \tan^{-1} (\omega CR)$

**Figure H.2** First-order all-pass filter.
| Filter Type and $T(s)$ | $s$-Plane Singularities | $|T|$ |
|------------------------|-------------------------|-------|
| (a) Low pass (LP)      | $T(s) = \frac{a_0}{s^2 + \omega_0 + \frac{\omega_0^2}{Q}}$ | $|T| = \frac{|a_0|Q}{\omega_0^2 \sqrt{1 - \frac{1}{4Q^2}}}$ |
|                        | DC gain $\frac{a_0}{\omega_0}$ | $\omega_{\text{max}} = \omega_0 \sqrt{1 - \frac{1}{2Q^2}}$ |
|                        | $\omega_{\text{max}} = \frac{\omega_0}{\sqrt{1 - \frac{1}{2Q^2}}}$ | $\omega_{\text{max}} = \frac{\omega_0}{\sqrt{1 - \frac{1}{2Q^2}}}$ |
|                        | $0 \rightarrow \infty$ | $0 \rightarrow \infty$ |

(b) High pass (HP)  
$T(s) = \frac{a_2s^2}{s^2 + \omega_0 + \frac{\omega_0^2}{Q}}$  
High-frequency gain $a_2$  

(c) Bandpass (BP)  
$T(s) = \frac{a_1s}{s^2 + \omega_0 + \frac{\omega_0^2}{Q}}$  
Center-frequency gain $\frac{a_1Q}{\omega_0}$  

Figure H.3 Second-order filter functions.
| Filter Type and $T(s)$ | s-Plane Singularities | $|T|\rightarrow$ |
|-------------------------|----------------------|-----------------|
| (d) Notch | $T(s) = a_2 \frac{s^2 + \omega_n^2}{s^2 + \frac{\omega_n^2}{Q} + \omega_0^2}$ | $|T| \sim \sqrt{2}$ |
| DC gain = High-frequency gain = $a_2$ | $\omega_1 \omega_2 = \omega_0^2$ |
| (e) Low-pass notch (LPN) | $T(s) = a_2 \frac{s^2 + \omega_n^2}{s^2 + \frac{\omega_n^2}{Q} + \omega_0^2}$ | $\omega_{\max} = \omega_0$ |
| DC gain = $\frac{\omega_n^2}{\omega_0^2}$ \(\omega_n \geq \omega_0\) | $\frac{\omega_n^2}{\omega_0^2} (1 - \frac{1}{2Q^2}) - 1$ |
| High-frequency gain = $a_2$ | $\frac{\omega_n^2}{\omega_0^2} + \frac{1}{2Q^2} - 1$ |
| (f) High-pass notch (HPN) | $T(s) = a_2 \frac{s^2 + \omega_n^2}{s^2 + \frac{\omega_n^2}{Q} + \omega_0^2}$ | $T_{\max} \rightarrow$ |
| DC gain = $\frac{\omega_n^2}{\omega_0^2}$ \(\omega_n \leq \omega_0\) | $\frac{\omega_n^2 - \omega_{\max}^2}{\sqrt{(\omega_0^2 - \omega_{\max}^2)^2 + \left(\frac{\omega_0}{Q}\right)^2}} \omega_{\max}$ |
| High-frequency gain = $a_2$ | $|T| \rightarrow |a_2|$ |

Figure H.3 continued
All pass (AP)

Flat gain $= a_2^2 T(s) = a_2^2 s^2 - 0$

Figure H.3 continued
Figure H.4 Realization of various second-order filter functions using the LCR resonator of Fig. 14.18(a): (a) general structure, (b) LP, (c) HP, (d) BP, (e) notch at \( \omega_0 \), (f) general notch, (g) LPN \( (\omega_n > \omega_0) \), (h) LPN as \( s \rightarrow \infty \), (i) HPN \( (\omega_n < \omega_0) \).
Figure H.5 Realization of the second-order all-pass transfer function using a voltage divider and an LCR resonator.
Figure H.6 Realizations for the various second-order filter functions using the op amp–RC resonator of Fig. 14.21(b): (a) LP, (b) HP, (c) BP. The circuits are based on the LCR circuit in Fig. 14.18. Design considerations are given in Table H.1.
Figure H.6 continued (d) Notch at $\omega_0$; (e) LPN, $\omega_n \geq \omega_0$; (f) HPN, $\omega_n \leq \omega_0$. 
Figure H.6 continued (g) All pass.

Table H.1 Design Data for the Circuits of Fig. H.6

<table>
<thead>
<tr>
<th>Circuit</th>
<th>Transfer Function and Other Parameters</th>
<th>Design Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resonator</td>
<td>[\omega_0 = \frac{1}{\sqrt{C_4 C_6 R_1 R_3 R_5}}]</td>
<td>[C_4 = C_6 = C \text{ (practical value)}] [R_1 = R_2 = R_3 = R_4 = \frac{1}{\omega_0 C}] [R_6 = Q/\omega_0 C]</td>
</tr>
<tr>
<td>Low-pass (LP)</td>
<td>[T(s) = \frac{K R_6/C_4 R_6 R_1 R_3}{s^2 + s R_2 + \frac{R_2}{C_6 R_6}}] [K = \text{DC gain}]</td>
<td></td>
</tr>
<tr>
<td>High-pass (HP)</td>
<td>[T(s) = \frac{K s^2}{s^2 + s \frac{1}{C_6 R_6} + \frac{R_2}{C_4 C_6 R_1 R_3 R_5}}] [K = \text{High-frequency gain}]</td>
<td></td>
</tr>
<tr>
<td>Bandpass (BP)</td>
<td>[T(s) = \frac{K s/C_6 R_6}{s^2 + s \frac{1}{C_6 R_6} + \frac{R_2}{C_4 C_6 R_1 R_3 R_5}}] [K = \text{Center-frequency gain}]</td>
<td></td>
</tr>
<tr>
<td>Regular notch (N)</td>
<td>[T(s) = K \left[\frac{s^2 + (R_2/C_4 C_6 R_1 R_3 R_5)}{s^2 + s \frac{1}{C_6 R_6} + \frac{R_2}{C_4 C_6 R_1 R_3 R_5}}\right]] [K = \text{Low- and high-frequency gain}]</td>
<td></td>
</tr>
</tbody>
</table>
### Low-pass Notch (LPN)

**Fig. H.6(e)**

\[
T(s) = K \frac{C_{61}}{C_{61} + C_{62}} \\
\times \frac{s^2 + \left(\frac{R_2}{C_4}C_{61}R_1R_3R_5\right)}{s^2 + s \left(\frac{C_{61} + C_{62}}{R_6} + \frac{C_4(C_{61} + C_{62})R_1R_3}{R_2}\right)}
\]

- \(K\) = DC gain
- \(C_{61} + C_{62} = C\)
- \(C_{61} = C(\omega_n/\omega_0)^2\)
- \(C_{62} = C - C_{61}\)

**Notch Frequency:**

\[
\omega_n = \frac{1}{\sqrt{\frac{C_4C_{61}R_1R_3}{R_2}}}
\]

**Gain Frequency:**

\[
\omega_0 = \frac{1}{\sqrt{\frac{C_4(C_{61} + C_{62})R_1R_3}{R_2}}}
\]

**Quality Factor:**

\[
Q = R_6 \sqrt{\frac{C_{61} + C_{62}}{C_4}} \frac{R_3}{R_1R_3}
\]

### High-pass Notch (HPN)

**Fig. H.6(f)**

\[
T(s) = K \frac{s^2 + \left(\frac{R_2}{C_4}C_{61}R_1R_3R_5\right)}{s^2 + s \left(\frac{C_{61} + C_{62}}{C_4}R_6 + \frac{C_4(C_{61} + C_{62})R_1R_3}{R_2}\right)}
\]

- \(K\) = High-frequency gain
- \(\frac{1}{R_{51}} + \frac{1}{R_{52}} = \frac{1}{R_2} = \omega_0 C\)
- \(R_{51} = R_5 (\omega_0/\omega_n)^2\)
- \(R_{52} = R_5 \left[1 - (\omega_0/\omega_n)^2\right]\)

**Notch Frequency:**

\[
\omega_n = \frac{1}{\sqrt{\frac{C_4C_{61}R_1R_3}{R_2}}}
\]

**Gain Frequency:**

\[
\omega_0 = \frac{1}{\sqrt{\frac{C_4(C_{61} + C_{62})R_1R_3}{C_4}}}
\]

**Quality Factor:**

\[
Q = R_6 \sqrt{\frac{C_{61} + C_{62}}{C_4}} \frac{R_3}{R_1R_3}
\]

### All-pass (AP)

**Fig. H.6(g)**

\[
T(s) = \frac{s^2 + \frac{1}{C_6R_0}r_2 + \frac{R_2}{C_6C_4R_1R_3}}{s^2 + s \left(\frac{R_2}{C_6R_0} + \frac{C_4C_6R_1R_3}{R_2}\right)}
\]

- \(r_1 = r_2 = r\) (arbitrary)
- \(r_2 = r\)
- Adjust \(r_2\) to make \(Q_z = Q\)

**Notch Frequency:**

\[
\omega_n = \omega_0
\]

**Gain Frequency:**

\[
Q_z = Q(r/r_2)
\]

**Flat Gain:**

\[
\text{Flat gain} = 1
\]
Figure H.7 The Tow–Thomas biquad with feedforward. The transfer function of Eq. (14.70) is realized by feeding the input signal through appropriate components to the inputs of the three op amps. This circuit can realize all special second-order functions. The design equations are given in Table H.2.

Table H.2 Design Data for the Circuit in Fig. H.7 (and Fig. 14.26)

<table>
<thead>
<tr>
<th>Type</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>All cases</td>
<td>$C$ = arbitrary, $R = 1/\omega_0 C$, $r$ = arbitrary</td>
</tr>
<tr>
<td>LP</td>
<td>$C_1 = 0$, $R_1 = \infty$, $R_2 = R/\text{dc gain}$, $R_3 = \infty$</td>
</tr>
<tr>
<td>Positive BP</td>
<td>$C_1 = 0$, $R_1 = \infty$, $R_2 = \infty$, $R_3 = Qr/\text{center-frequency gain}$</td>
</tr>
<tr>
<td>Negative BP</td>
<td>$C_1 = 0$, $R_1 = QR/\text{center-frequency gain}$, $R_2 = \infty$, $R_3 = \infty$</td>
</tr>
<tr>
<td>HP</td>
<td>$C_1 = C \times \text{high-frequency gain}$, $R_1 = \infty$, $R_2 = \infty$, $R_3 = \infty$</td>
</tr>
<tr>
<td>Notch</td>
<td>$C_1 = C \times \text{high-frequency gain}$, $R_1 = \infty$, $R_2 = R(\omega_0/\omega_n)^2/\text{high-frequency gain}$, $R_3 = \infty$</td>
</tr>
<tr>
<td>AP</td>
<td>$C_1 = C \times \text{flat gain}$, $R_1 = \infty$, $R_2 = R/\text{gain}$, $R_3 = Qr/\text{gain}$</td>
</tr>
</tbody>
</table>