APPENDIX D

SOME USEFUL NETWORK THEOREMS

Introduction

In this appendix we review three network theorems that are useful in simplifying the analysis of electronic circuits: Thévenin’s theorem, Norton’s theorem, and the source-absorption theorem.

D.1 Thévenin’s Theorem

Thévenin’s theorem is used to represent a part of a network by a voltage source $V_t$ and a series impedance $Z_t$, as shown in Fig. D.1. Figure D.1(a) shows a network divided into two parts, A and B. In Fig. D.1(b), part A of the network has been replaced by its Thévenin equivalent: a voltage source $V_t$ and a series impedance $Z_t$. Figure D.1(c) illustrates how $V_t$ is to be determined: Simply open-circuit the two terminals of network A and measure (or calculate) the voltage that appears between these two terminals. To determine $Z_t$, we reduce all external (i.e., independent) sources in network A to zero by short-circuiting voltage sources and open-circuiting current sources. The impedance $Z_t$ will be equal to the input impedance of network A after this reduction has been performed, as illustrated in Fig. D.1(d).

D.2 Norton’s Theorem

Norton’s theorem is the dual of Thévenin’s theorem. It is used to represent a part of a network by a current source $I_n$ and a parallel impedance $Z_n$, as shown in Fig. D.2. Figure D.2(a) shows a network divided into two parts, A and B. In Fig. D.2(b), part A has been replaced by its Norton’s equivalent: a current source $I_n$ and a parallel impedance $Z_n$. The Norton’s current source $I_n$ can be measured (or calculated) as shown in Fig. D.2(c). The terminals of the network being reduced (network A) are shorted, and the current $I_n$ will be equal simply to the short-circuit current. To determine the impedance $Z_n$, we first reduce the external excitation in network A to zero: That is, we short-circuit independent voltage sources and open-circuit independent current sources. The impedance $Z_n$ will be equal to the input impedance of network A after this source-elimination process has taken place. Thus the Norton impedance $Z_n$ is equal to the Thévenin impedance $Z_t$. Finally, note that $I_n = V_t/Z$, where $Z = Z_n = Z_t$. 
Some Useful Network Theorems

Figure D.1 Thévenin’s theorem.

Figure D.2 Norton’s theorem.

Example D.1

Figure D.3(a) shows a bipolar junction transistor circuit. The transistor is a three-terminal device with the terminals labeled E (emitter), B (base), and C (collector). As shown, the base is connected to the dc power supply $V^+$ via the voltage divider composed of $R_1$ and $R_2$. The collector is connected to the dc supply $V^+$ through $R_1$ and to ground through $R_2$. To simplify the analysis, we wish to apply Thévenin’s theorem to reduce the circuit.

Solution

Thévenin’s theorem can be used at the base side to reduce the network composed of $V^+$, $R_1$, and $R_2$ to a dc voltage source $V_{BB}$,

$$V_{BB} = V^+ \frac{R_2}{R_1 + R_2}$$
Figure D.3 Thévenin’s theorem applied to simplify the circuit of (a) to that in (b). (See Example D.1.)

and a resistance $R_B$,

$$R_B = R_1 \parallel R_2$$

where $\parallel$ denotes “in parallel with.” At the collector side, Thévenin’s theorem can be applied to reduce the network composed of $V^+$, $R_3$, and $R_4$ to a dc voltage source $V_{CC}$,

$$V_{CC} = V^+ \frac{R_4}{R_3 + R_4}$$

and a resistance $R_C$,

$$R_C = R_3 \parallel R_4$$

The reduced circuit is shown in Fig. D.3(b).

### D.3 Source-Absorption Theorem

Consider the situation shown in Fig. D.4. In the course of analyzing a network, we find a controlled current source $I_x$ appearing between two nodes whose voltage difference is the controlling voltage $V_x$. That is, $I_x = g_m V_x$ where $g_m$ is a conductance. We can replace this controlled source by an impedance $Z_x = V_x/I_x = 1/g_m$, as shown in Fig. D.4, because the current drawn by this impedance will be equal to the current of the controlled source that we have replaced.

![Figure D.4](image-url)  
**Figure D.4** The source-absorption theorem.
Example D.2

Figure D.5(a) shows the small-signal, equivalent-circuit model of a transistor. We want to find the resistance \( R_{\text{in}} \) “looking into” the emitter terminal E—that is, the resistance between the emitter and ground—with the base B and collector C grounded.

\[ R_{\text{in}} = r_{\pi} \parallel \left( \frac{1}{g_m} \right) \]

Figure D.5 Circuit for Example D.2.

Solution

From Fig. D.5(a), we see that the voltage \( v_e \) will be equal to \(-v_i\). Thus, looking between E and ground, we see a resistance \( r_{\pi} \) in parallel with a current source drawing a current \( g_m v_e \) away from terminal E. The latter source can be replaced by a resistance \( \frac{1}{g_m} \), resulting in the input resistance \( R_{\text{in}} \) given by

\[ R_{\text{in}} = r_{\pi} \parallel \left( \frac{1}{g_m} \right) \]

as illustrated in Fig. D.5(b).

EXERCISES

D.1 A source is measured and found to have a 10-V open-circuit voltage and to provide 1 mA into a short circuit. Calculate its Thévenin and Norton equivalent source parameters.

\[ \text{Ans. } V_t = 10 \text{ V}; Z_t = Z_n = 10 \text{ k}\Omega; I_n = 1 \text{ mA} \]

D.2 In the circuit shown in Fig. ED.2, the diode has a voltage drop \( V_D \simeq 0.7 \text{ V} \). Use Thévenin’s theorem to simplify the circuit and hence calculate the diode current \( I_D \).

\[ \text{Ans. } 1 \text{ mA} \]
D.3 The two-terminal device M in the circuit of Fig. ED.3 has a current \( I_M \approx 1 \text{ mA} \) independent of the voltage \( V_M \) across it. Use Norton’s theorem to simplify the circuit and hence calculate the voltage \( V_M \).

**Ans.** 5 V

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**Problems**

**D.1** Consider the Thévenin equivalent circuit characterized by \( V_t \) and \( Z_t \). Find the open-circuit voltage \( V_{oc} \) and the short-circuit current \( I_{sc} \) (i.e., the current that flows when the terminals are shorted together). Express \( Z_t \) in terms of \( V_{oc} \) and \( I_{sc} \).

**D.2** Repeat Problem D.1 for a Norton equivalent circuit characterized by \( I_n \) and \( Z_n \).

**D.3** A voltage divider consists of a 9-kΩ resistor connected to +10 V and a resistor of 1 kΩ connected to ground. What is the Thévenin equivalent of this voltage divider? What output voltage results if it is loaded with 1 kΩ? Calculate this two ways: directly and using your Thévenin equivalent.
**D.4** Find the output voltage and output resistance of the circuit shown in Fig. PD.4 by considering a succession of Thévenin equivalent circuits.

**D.5** Repeat Example D.2 with a resistance $R_B$ connected between B and ground in Fig. D.5 (i.e., rather than directly grounding the base B as indicated in Fig. D.5).

**D.6** Figure PD.6(a) shows the circuit symbol of a device known as the $p$-channel junction field-effect transistor (JFET). As indicated, the JFET has three terminals. When the gate terminal G is connected to the source terminal S, the two-terminal device shown in Fig. PD.6(b) is obtained. Its $i$–$v$ characteristic is given by

\[
\begin{align*}
  i &= I_{DSS} \left[ 1 + \frac{v}{V_P} \right] \quad \text{for } v \leq V_P \\
  i &= I_{DSS} \quad \text{for } v \geq V_P
\end{align*}
\]

where $I_{DSS}$ and $V_P$ are positive constants for the particular JFET. Now consider the circuit shown in Fig. PD.6(c) and let $V_P = 2\,\text{V}$ and $I_{DSS} = 2\,\text{mA}$. For $V^+ = 10\,\text{V}$ show that the JFET is operating in the constant-current mode and find the voltage across it. What is the minimum value of $V^+$ for which this mode of operation is maintained? For $V^+ = 2\,\text{V}$ find the values of $I$ and $V$. 

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**Figure PD.4**

**Figure PD.6**