The Golden Section

The study of composition often begins as theory in your two-dimensional design courses and should continue to be developed and reinforced in an applied sense throughout your fine arts education. Along with informal systems and principles of organization, it is instructive and informative to investigate the various formal systems of organization that have withstood close scrutiny and the test of time. The most prominent of these is the Golden Section. Examples from art and nature showing the application and presence of the Golden Section indicate the prevalence and significance of this aesthetic device and the underlying organization found in seemingly random or chaotic structures. Although it can be a bit overwhelming to the uninitiated with its strong foothold in mathematics, Euclidian geometry, and

Pythagorean theory, proficiency in these subjects is not necessary to a basic understanding of the Golden Section and its role in nature and the arts. If you find your interest is piqued and you wish to investigate further, a number of resources are listed in the general Bibliography and in the Supplemental Reading for Design Principles. Following is a basic discussion of the Golden Section.

WHAT IS THE GOLDEN SECTION?

The Golden Section (also known as the Golden Mean, the Golden Proportion, the Golden Ratio, the Golden Rectangle, and the Divine Proportion) is a system of aesthetically pleasing proportions based on the division of space into parts that correspond to the proportion of .618 to 1 or 1 to 1.618. An *approximation* of this proportion is expressed as three parts to five parts (3:5) or five parts to eight parts (5:8). Expressed differently, the smaller part relates to the size of the greater part in the same way that the greater part relates to the size of the whole (the sum of both the smaller and greater part). Numerically, this can be expressed *approximately* as 3 parts relate to 5 parts in the same way as 5 parts relate to 8 parts. On any given line, there is only one point that will bisect it into two unequal parts in this uniquely reciprocal fashion, and this one point is called the point of Golden Section (Figure 1-61). In fractions, the Golden Section can be generally expressed as a ⅗ or ⅝ division of space, but this is only an approximation that is used for the sake of convenience. This principle can be applied to a line, to a two-dimensional plane, or to a three-dimensional solid, and it is the only proportional relationship to increase by geometric progression and by simple addition simultaneously. This "divine" proportion is credited by some with various mystical properties and exceptional beauties both in science and in art.

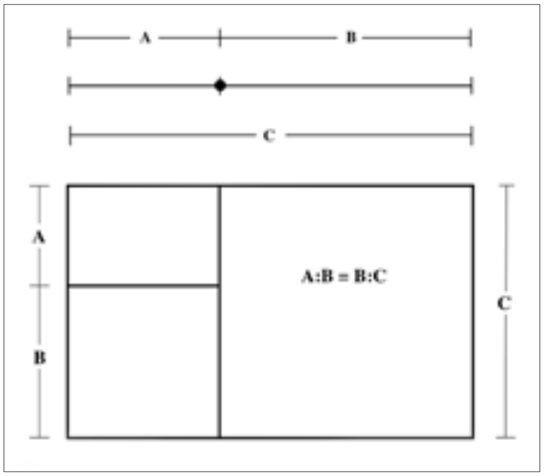


Figure 1-61. The Golden Section. A rectangle whose dimensions reflect the Golden Section is subdivided vertically and horizontally at points of Golden Section to create yet another golden rectangle. This subdivision can recur over and over again, creating increasingly smaller golden rectangles. This point of division can be applied to a line, to a rectangular plane, or to a three-dimensional solid.

Evidence of the principle of the Golden Section is found in the human body, in the capital letters of the Latin alphabet, in various forms throughout nature (including the structural formations of DNA, certain viruses, and quasi-crystals), and in architectural structures and details (Figure 1-62). Based on a Euclidean theory, Vitruvius worked out the Golden Section in the first century BCE to establish architectural standards for the proportions of columns, rooms, and entire buildings, with the understanding that individual variations were expected of the architect. Historical masters such as Leonardo da Vinci, Piero Della Francesca, and Leon Battista Alberti all were aware of the significance of the Golden Section, and evidence of its application can be found throughout their work. More recently, the Golden Section provided inspiration for the cubists and was employed by Georges Seurat, Juan Gris, Pablo Picasso, and other renowned artists.

Statistical experiments are said to have shown that human beings involuntarily give preference to proportions that approximate the Golden Section. In an October 1985 newspaper article distributed by the Associated Press and printed in the *Grand Rapids Press*, the following observations were made:

When a newborn baby first identifies its mother's face, it's not by color, shape, smell or heat, but the way the eyebrows, eyes, nose and mouth are arranged in proportions artists have known for centuries as the "golden ratio," says a noted child psychiatrist.

"It seems clear that nature has built a certain sense of proportions into the human organism that have survival value," said Dr. Eugene J. Mahon. "In other words, because it's essential for children to get to know the imprint of their mother's face, nature has not left it to chance."

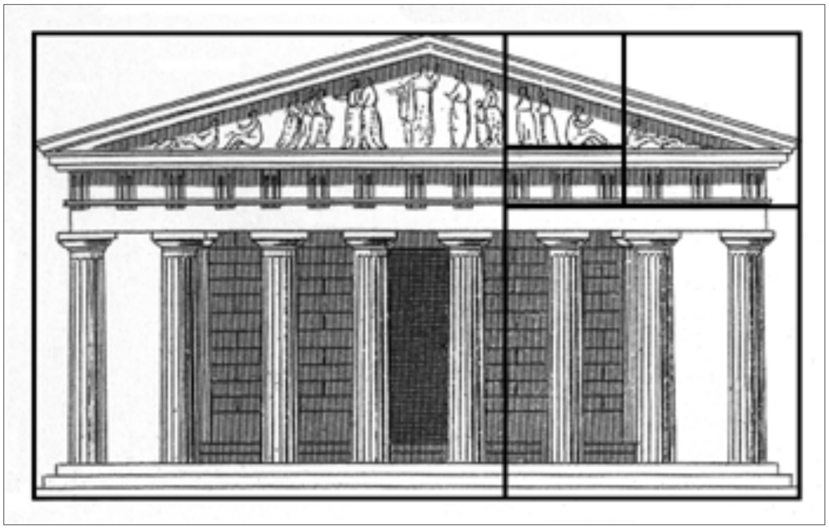


Figure 1-62. Eastern facade of the Parthenon. Courtesy of Dover Pictorial Archive Series, Dover Publications, Inc. This illustration superimposes an approximation of the Golden Section rectangle on the eastern facade of the Parthenon, showing the presence of the Golden Section proportions in the entire facade as well as in the entablature and pediment.

Those proportions, Mahon said, happen to be the same as the famous proportion known for centuries by artists and architects as the "golden ratio," or "golden section." The golden ratio has been proven by experiments as particularly harmonious and pleasing to the human senses. It is not surprising, therefore, if it catches even the untrained eyes of a newborn, he said.

The Irish psychiatrist, who is on the faculty of Columbia University's Psychoanalytic Clinic for Training and Research as well as the Columbia College of Physicians and Surgeons, presented his findings in The Psychoanalytic Study of the Child.

"A mask constructed with two eyes, a nose and a mouth enclosed in an oval that resembles the human face will hold the attention of the infant as much as the human face itself," Mahon said in an interview, citing his and others' experiments.

"A mask with features out of proportion to expectable human anatomy, or the human face itself in profile, does not attract the attention of the infant nearly as intently as a human face. The distance from the top of the human head to the eyebrows and the distance from the eyebrows to the end of the chin are roughly 5 and 8, no matter what units of measurements are used. The distance from the tip of the nose to the lips and the distance from the lips to the end of the chin also relate to each other as 5 and 8. In the human face and body, we can find many combinations of the golden section."

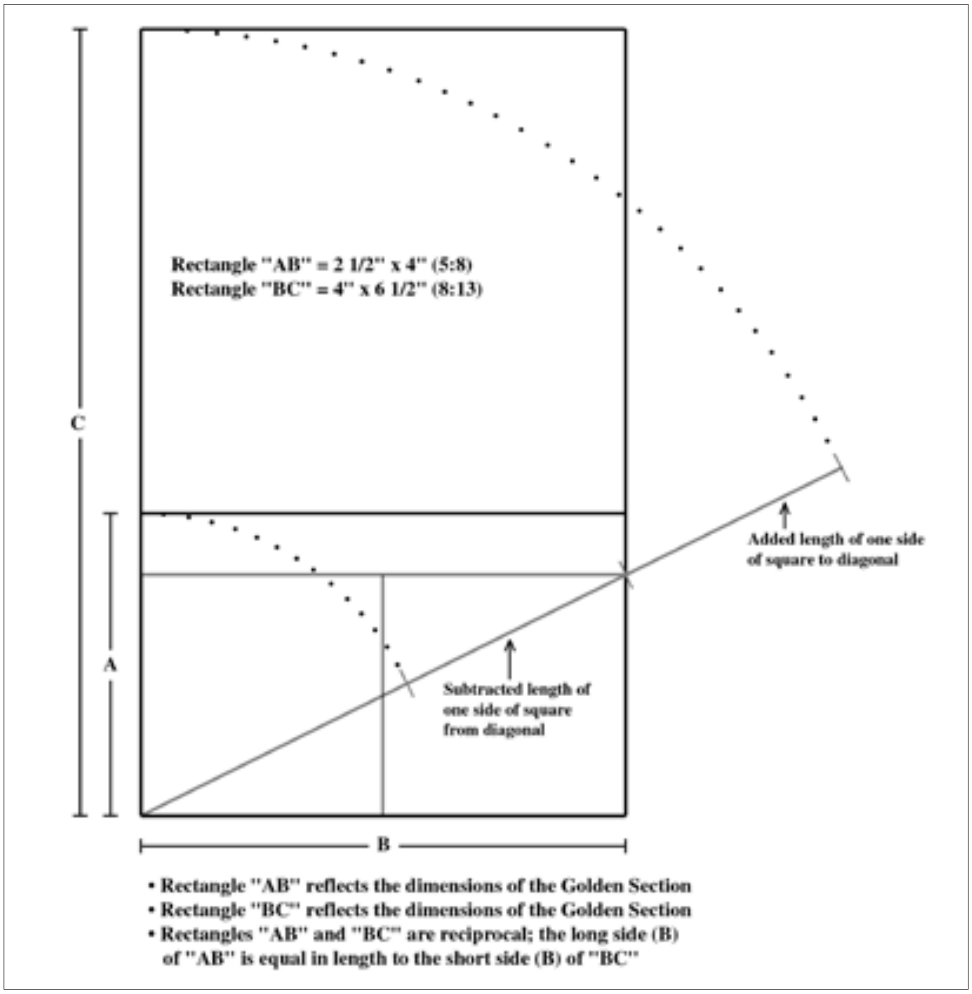
The human preference for this particular proportion, which abounds in nature, architecture and classical paintings, has been known through the ages, the psychoanalyst said. Examples of golden rectangles are found on paper money, traveler's checks and credit cards, he said.

CONSTRUCTING A GOLDEN RECTANGLE

The "ideal" proportions of a rectangle are determined by the Golden Section, which explores projections of a square within itself and outside of itself to create pleasing and harmonious proportional relationships. Geometrically, the Golden Section may be constructed by means of the diagonal of a rectangle composed of two squares (Figure 1-63). From the diagonal of a rectangle composed of two squares (1×2), add or subtract the length of one side of the square and place the resulting length against (at a 90° angle to) the longer side of the original rectangle. *Adding* the length of one side of the square creates the longer side of the resulting Golden Section rectangle (BC). *Subtracting* the length of one side of the square creates the shorter side of the resulting Golden Section rectangle (AB). The ratio of the two sides of the rectangle works out numerically to .618 to 1, or approximately 5 to 8.

A second method for constructing a Golden Section rectangle is based on the projection of a square outside of itself (Figure 1-64). Starting with a square, find the center point of the base of the square and from this point draw a line to the upper left or right corner of the square. Rotate this line left or right from the center point of the base of the square until it aligns with and extends the base of the square. Constructing an additional vertical and horizontal line from this point of extension defines the proportions of a Golden Section rectangle. The Golden Section spiral results from rotating the length of one side of the square within each of the series of squares that result from the repeated construction of successively smaller Golden Section

Figure 1-63. Construction of a Golden Section rectangle based on the diagonal of a rectangle composed of two squares.



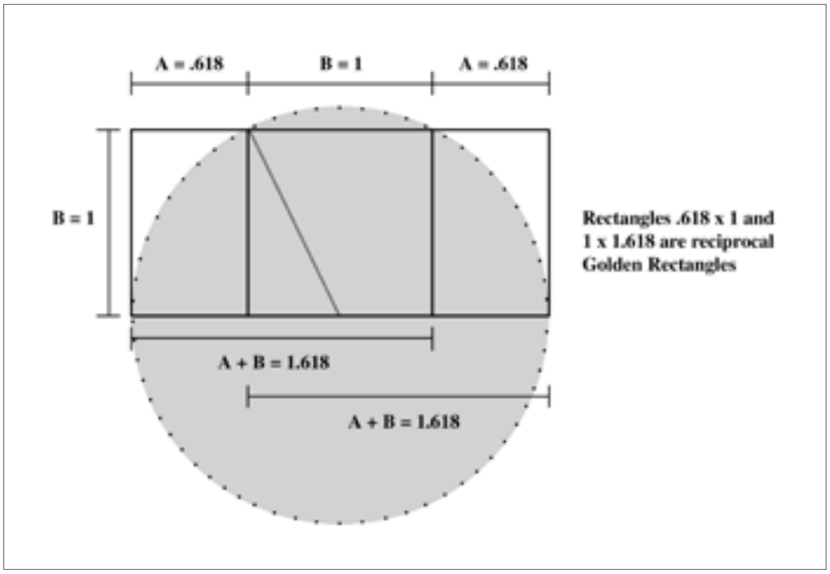


Figure 1-64. Construction of a Golden Section rectangle based on the projection of a square outside of itself.

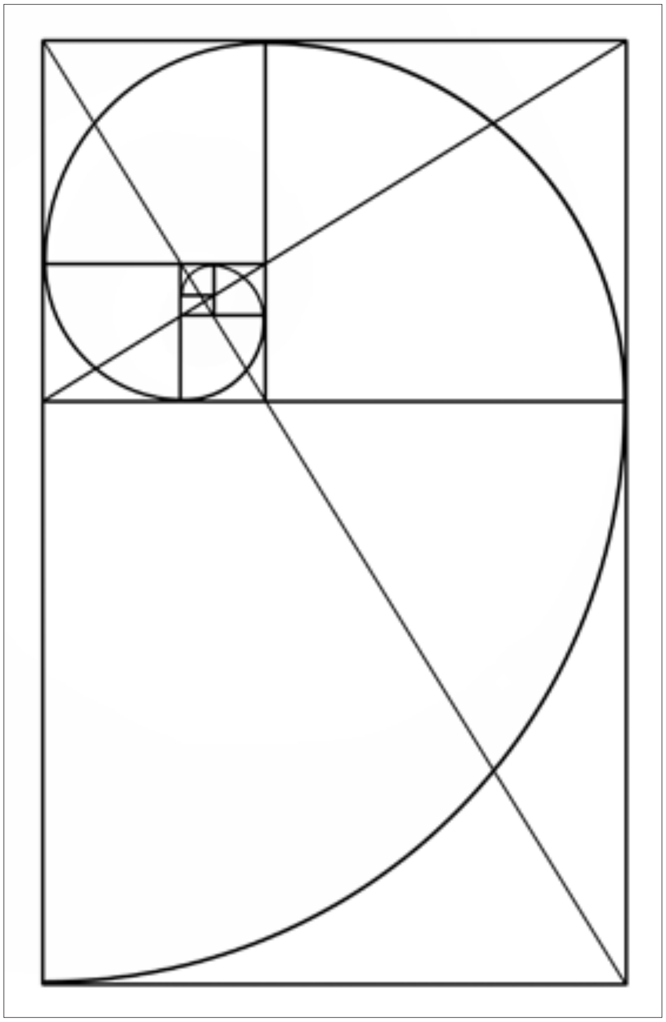


Figure 1-65. A Golden Section spiral. The spiral is created by rotating the length of the side of each square, with the squares becoming increasingly smaller. Theoretically, this process can continue indefinitely. Notice that the two diagonals, which cross at right angles at the point of Golden Section, are the diagonals for every Golden Section rectangle, both large and small, in the diagram.

rectangles (Figure 1-65). The spiral, which is theoretically infinite, ultimately focuses on a point defined by the intersection of the diagonals of both horizontally and vertically oriented Golden Section rectangles. This is called the point of Golden Section.

A rectangle that reflects the proportions of the Golden Section has four points of Golden Section, regardless of the orientation of the rectangle (Figure 1-66). But any rectangular shape also provides four points of Golden Section based on the diagonal of the rectangle intersected at right angles by a line extended from each of the four corners (Figure 1-67).

The Golden Section is unique in that its mathematical equivalent represents the only instance in which a ratio is the same or equal to a proportion. In other words, the expression of the Golden Section as a fraction is equal to the expression of the Golden Section as a ratio (Figure 1-68). It is often claimed that the Golden Section is aesthetically superior to all other proportions. A significant amount of research and data supporting this claim has been collected over the years, from both the scientific community and from the arts community. Some resources that provide in-depth information on particular applications or instances of the Golden Section and other formal systems of organization can be found in the Bibliography and in the "Supplemental Reading for Design Principles" section of the Bibliography.

THE FIBONACCI SERIES

The Fibonacci Series is a system of expanding size relationships closely connected with the Golden Section. In mathematics, the Fibonacci Series is known

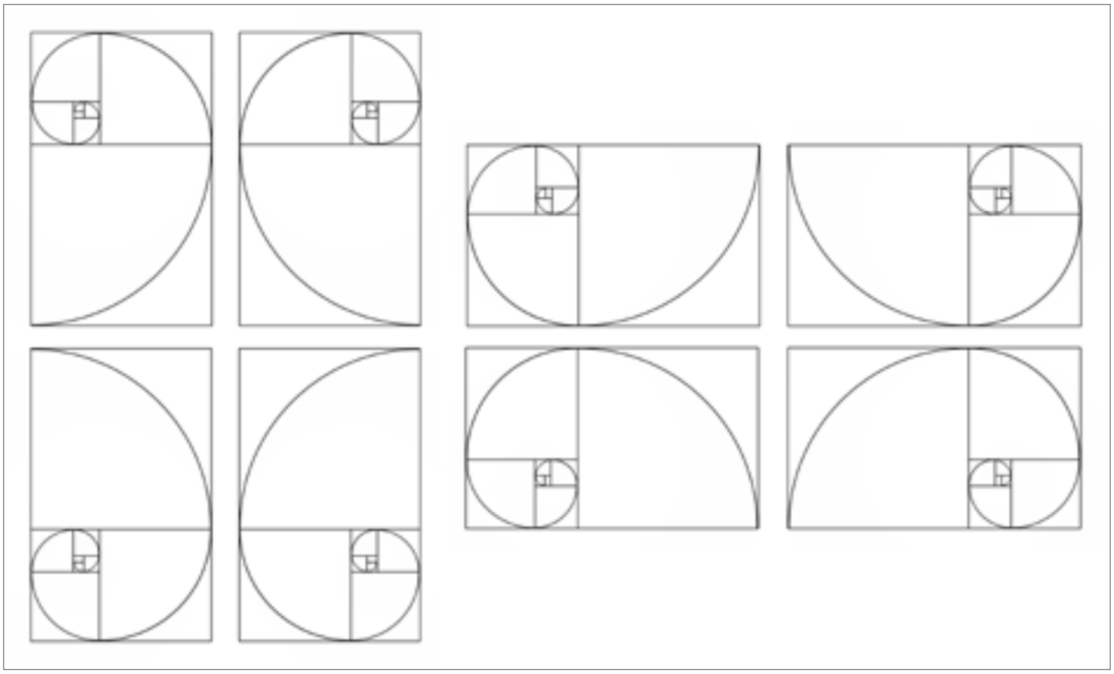
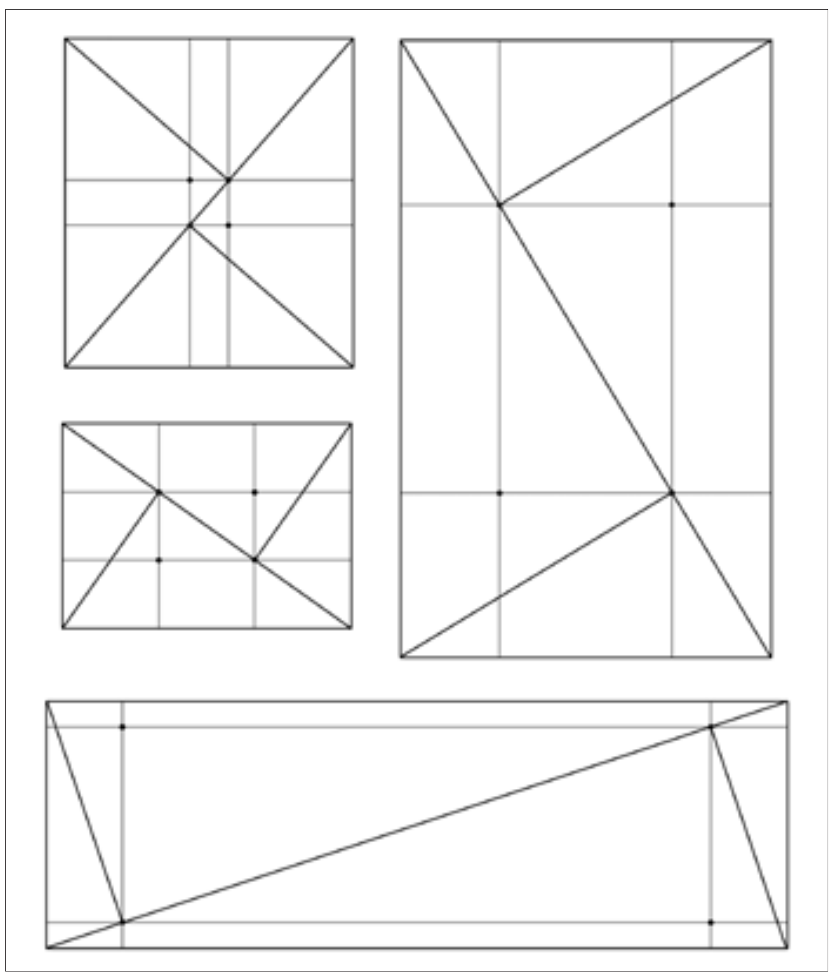


Figure 1-66. Every Golden Section rectangle, whether vertical or horizontal in orientation, has four points of Golden Section.

Figure 1-67. Rectangles that do not reflect the proportions of the Golden Section also have four points of Golden Section.



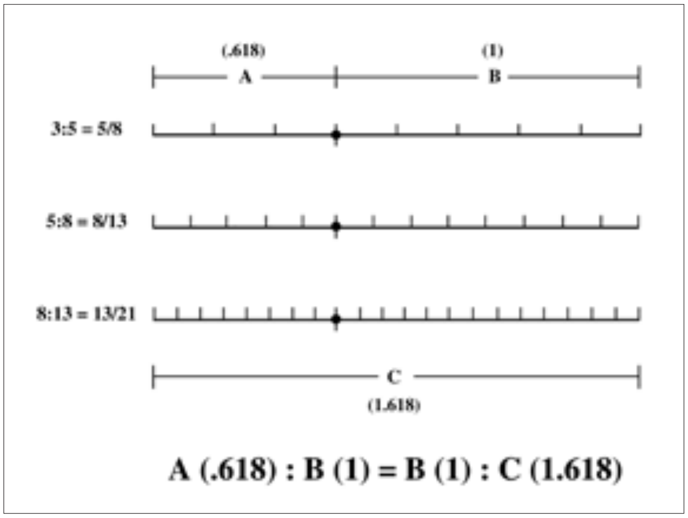


Figure 1-68. The point of Golden Section on a line is expressed equally as a fraction and a ratio. For example, the fraction of 5⁄8 essentially represents the same point of division as the ratio 5:8. Similarly, the fraction of 8⁄13 represents the same point of division as the ratio 8:13. This illustrates the relationship between the point of Golden Section and the Fibonacci Series. These are approximations rather than precise expressions because the earlier fractions and ratios in the Fibonacci sequence are less precise reflections of the Golden Section.

as a summation series in which each number is the sum of the two previous numbers. In the twelfth century, Leonardo of Pisa, also known as Leonardo Fibonacci (1175–1230), discovered that if a ladder of whole numbers is constructed so that each number on the right is the sum of the pair on the preceding rung, the arithmetical ratio between the two numbers on the same rung rapidly approaches the Golden Section (.618 to 1 or 1 to 1.618, or phi). Beginning with 0 and 1, the first two whole numbers, each subsequent number in the series is simply the sum of the previous two. This numerical sequence, carried out a few times, yields the following series of numbers, known as the Fibonacci Series: (0, 1, 1,) 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657 . . .

Again, the Fibonacci Series (shown in bold) results from the simple addition of each number and the previous number: **0** + **1** = **1**, 1 + 1 = **2**, 1 + 2 = **3**, 2 + 3 = **5**, 3 + 5 = **8**, 5 + 8 = **13**, 8 + 13 = **21**, 13 + 21 = **34**, 21 + 34 = **55**, 34 + 55 = **89**, 55 + 89 = **144**, 89 + 144 = **233**, 144 + 233 = **377**, 233 + 377 = **610**, 377 + 610 = **987**, 610 + 987 = **1597**, 987 +1597 = **2584**, 1597 + 2584 = **4181**, 2584 + 4181 = **6765**, 4181 + 6765 = **10946**, 6765 + 10946 = **17711**, 10946 + 17711 = **28657**.

The ratio of each successive pair of numbers in the series (2:3, 3:5, 5:8, 8:13, 13:21, etc.) increasingly approximates and converges on phi (1.618 . . .). Beginning with 2, any number in this series divided by the *following* one (e.g., 5 divided by 8) approximates .618, while any number in this series divided by the *previous* one (e.g., 8 divided by 5) approximates 1.618 (see Figure 1-68). These are the proportional rates between minor and major parts of the Golden Section: .618 (A) is to 1 (B) as 1 (B) is to 1.618 (C), or A:B = B:C. Note that the expression of the Golden Section as a fraction (5⁄8 or 8⁄13) approximates the same point of division as the expression of the Golden Section as a ratio (5:8 or 8:13). As you move higher up the ladder of numbers, this relationship becomes increasingly precise.

Further examples of a number divided by the *previous* number in the series include 3 divided by 2 equals 1.5, 5 divided by 3 equals 1.666 . . . , 8 divided by 5 equals 1.6, 13 divided by 8 equals 1.625, and, further along in the series, 2584 divided by 1597 equals 1.6180338134. Further examples of a number divided by the *following* number in the series include 2 divided by 3 equals .66666 . . . , 3 divided by 5 equals .6, 5 divided by 8 equals .625, 8 divided by 13 equals .6153846 . . . , and, further along in the series, 6765 divided by 10946 equals .6180339 . . . . After the 40th number in the series, the ratio is accurate up to fifteen decimal places.

In the study of phyllotaxis, which is the spiraling arrangement of leaves, scales, and flowers, it has been found that fractions representing these spiral arrangements are often members of the Fibonacci Series, which supports the notion that numerous instances of the Golden Section can be found in nature. One of the more striking examples of the Fibonacci Series in nature is the sunflower. Botanists have discovered that the average-size sunflower head, whose seeds are arranged in opposing spirals, shows exactly 55 seeds in a clockwise direction and exactly 89 seeds in a counterclockwise direction. Smaller and larger sunflower heads show the same ratio of opposing seed spirals such as 34 to 55 seed spirals, and 89 to 144 seed spirals. Nautilus shells, apple blossoms, daisies, artichokes, pussy willows, pineapple skins, and pinecones show similar ratios in their natural growth patterns. The proportions of the human body conform in many ways to this ratio.