

Baronett's *Logic* (4th ed.)
Section Tips

Chapter 7: Three Tips

7C Truth Functions

- The **truth value** of a compound proposition using one or more of the five operators is uniquely determined by the truth values of its component propositions:
 - Tilde is false when and *only* when the statement it negates is true. Tilde is true when the statement it negates is false.
 - Dot is true when and *only* when both sides of dot are true; otherwise, it is false.
 - Wedge is false when and *only* when both sides of wedge are false; otherwise it is true.
 - Horseshoe is false when and *only* when the antecedent is true and the consequent is false; otherwise, it is true.
 - Triple bar is true when and *only* when the truth values of each side are the same.

7D Truth Tables for Propositions

- Work from the inside out: begin with the truth values of the simple statements and determine the truth values of the least complicated compound statement, all the way out to the main operator. For example, the truth value of the statement, $\sim [A \supset \sim (B \vee C)]$ is worked out as follows:
 1. Determine the truth value of " $B \vee C$ " according to the truth definitions of " \vee ."
 2. Determine the truth value of " $\sim (B \vee C)$ " according to the truth definitions of " \sim ." Notice that in this statement, " \sim " is a denial of " \vee ."
 3. Determine the truth value of " $A \supset \sim (B \vee C)$ " according to the truth definitions of " \supset ." Notice that " \supset " is determined by comparing the values of the antecedent, " A " with the values of the consequent, " \sim ."
 4. Finally, determine the value of the main operator, \sim , by negating the value of the " \supset " within the brackets.

7H Indirect Truth Tables

- The indirect proof method of testing for validity nicely reveals the principle of noncontradiction at work in a valid argument. When we assume the premises are true and the conclusion is false, but the argument is valid, we run into a contradiction:

Assume the following:

$$\frac{\sim (A \vee B)}{T} \quad / \quad \frac{\sim A}{F}$$

Assuming that “ $\sim A$ ” is false, then “ A ” is true. Now, since we assumed that “ A ” is true, the wedge is true because at least one of the disjuncts is true. In this case, $\sim (A \vee B)$ is false. In other words, you can't have “ \vee ” be true and $\sim (A \vee B)$ true *at the same time*.

If the conclusion is false, then the premise cannot be true:

$$\frac{\sim (A \vee B)}{\cancel{T}} \quad / \quad \frac{\sim A}{F}$$

F

The argument is therefore *valid*.