Chapter 8: Natural Deduction

A. Natural Deduction

Natural deduction is a proof procedure based on the truth definitions of the logical operators, ~, •, v, ⊃, and ≡. This system uses **implication rules** and **replacement rule**s to derive the conclusion of a valid argument.

B. Implication Rules I

Implication rules are valid argument forms. They are validly applied only to an entire line of a proof.

Modus Ponens (MP), *Modus Tollens* (MT), **Hypothetical Syllogism** (HS), and **Disjunctive Syllogism** (DS) make up the first four implication rules in the system of natural deduction:

C. Tactics and Strategy

Natural deduction allows for creativity because sometimes more than one equally correct proof is possible for a given problem. This means that it is important to approach a proof with constructive ideas about how to reach the goal of deriving the conclusion from the premises.

Strategy is the overall approach you take to setting up a proof, such as initially locating the conclusion somewhere "inside" the premises. **Tactics** are small scale maneuvers as you work your way through a proof, using each of the rules.

D. Implication Rules II

Simplification (Simp), **Conjunction** (Conj), **Addition** (Add) and **Constructive Dilemma** (CD) make up the second four implication rules in the system of natural deduction:

E. Replacement Rules I

Both sets of replacements rules enlist the **principle of replacement**, which states that logically equivalent expressions may replace each other within the context of a proof. Replacement rules can be used either on a part of a line or an entire line in a proof.

De Morgan (DM), **Double Negation** (DN), **Commutation** (Com), **Association** (Assoc), **Distribution** (Dist) and are sets of equivalent statements, one of which can be substituted for the other in a proof.

F. Replacement Rules II

Transposition (Trans), **Material Implication** (Impl), **Material Equivalence** (Equiv), **Exportation** (Exp), and **Tautology** (Taut) make up the last sets of replacement rules.

G. Conditional Proof

Conditional Proof (CP) A method that starts by assuming the antecedent of a conditional statement on a separate line and then proceeds to validly derive the consequent on a separate line.

As with all assumptive proofs, the assumption must be "discharged"; you must get out of the subproof sequence and back onto the main line. This is accomplished by achieving the goal of the subproof.

H. Indirect Proof

Indirect Proof (IP) A method that starts by assuming the negation of the required statement and then validly deriving a contradiction on a subsequent line.

As with all assumptive proofs, the assumption must be "discharged"; you must get out of the subproof sequence and back onto the main line. This is accomplished by achieving the goal of the subproof.